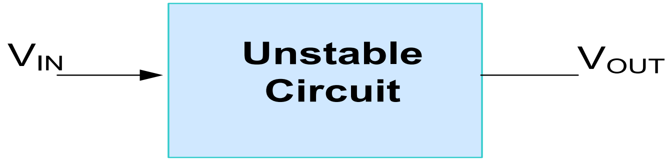
# EE 508
# Lecture 5

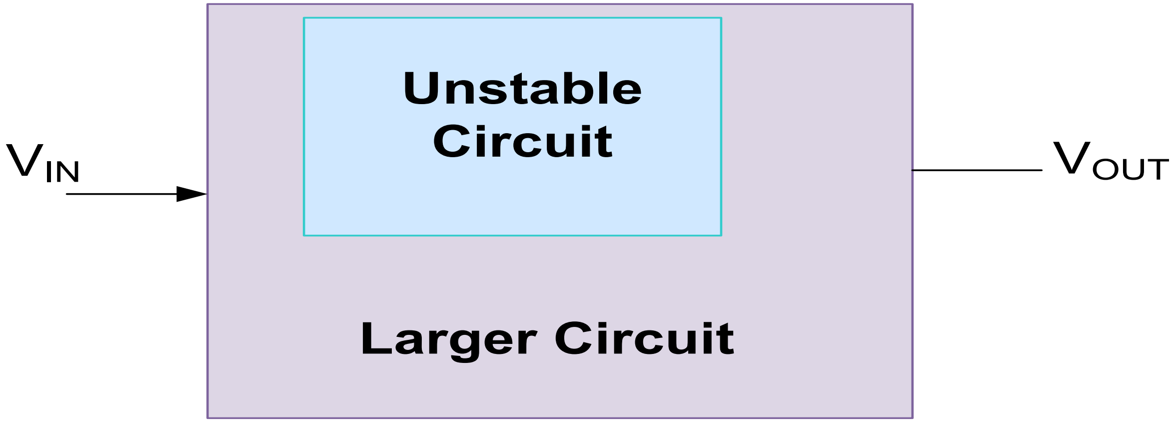
- Dead Networks
- Root Characterizations
- Scaling, Normalization and Transformations
- Degrees of Freedom and Systematic Design

## Review from Last Time

**Theorem ?:**

If a circuit is unstable, then if this circuit is included as a subcircuit in a larger circuit structure, the larger circuit will also be unstable.



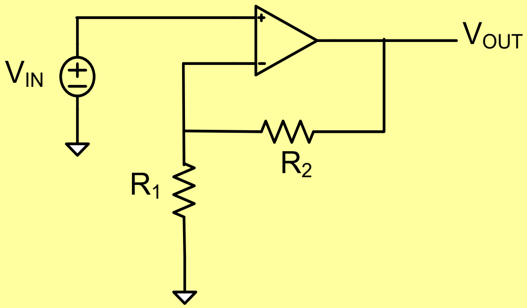

Proof:

**This theorem is not valid though many circuit and filter designers believe it to be true !**

Review from Last Time

# Gain, Bandwidth and GB

## Summary of Effects of GB on Basic Inverting and Noninverting Amplifiers

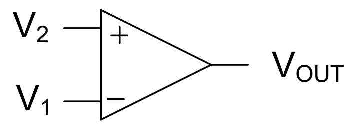



Basic Noninverting Amplifier

$$K_0 = 1 + \frac{R_2}{R_1}$$

$$BW = \frac{GB}{K_0}$$

$$A_{FB}(s) = \frac{K_0}{1 + s\frac{K_0}{GB}}$$

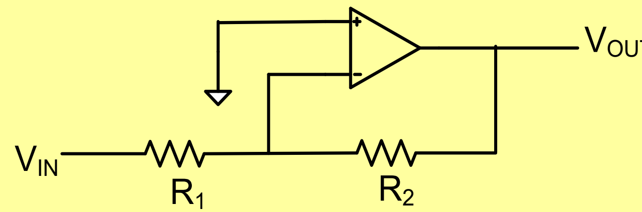



$$A_1(s) = \frac{GB}{s + BW_A}$$

$$GB = A_0 \bullet BW_A$$

$$A(s) = \frac{GB}{s}$$

Adequate model for most applications

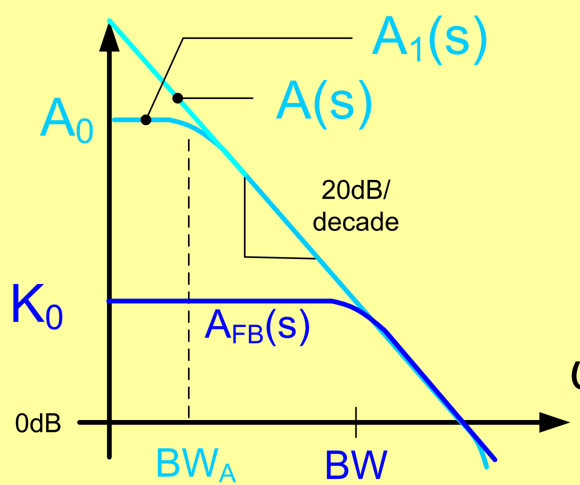



Basic Inverting Amplifier

$$K_0 = \frac{R_2}{R_1}$$

$$BW = \frac{GB}{1 + K_0}$$

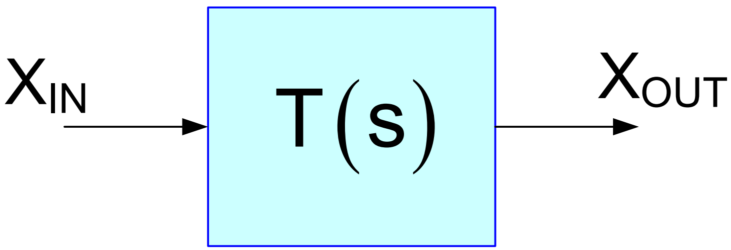
$$A_{FB}(s) = -\frac{K_0}{1 + s\frac{(1+K_0)}{GB}}$$



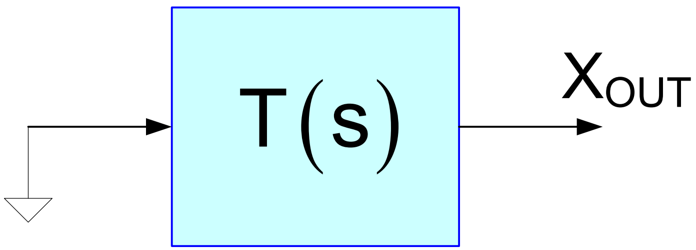
# Filter Concepts and Terminology

- 2-nd order polynomial characterization
- Biquadratic Factorization
- Op Amp Modeling
- Stability and Instability
- Roll-off characteristics
- Distortion
- Dead Networks
- Root Characterization
- Scaling, normalization, and transformation

# Dead Networks




$$T(s) = \frac{N(s)}{D(s)}$$




$D(s)$

The "dead network" of any linear circuit is obtained by setting ALL independent sources to zero.

- Replace independent current sources with opens
- Replace independent voltage sources with shorts
- Dependent sources remain intact

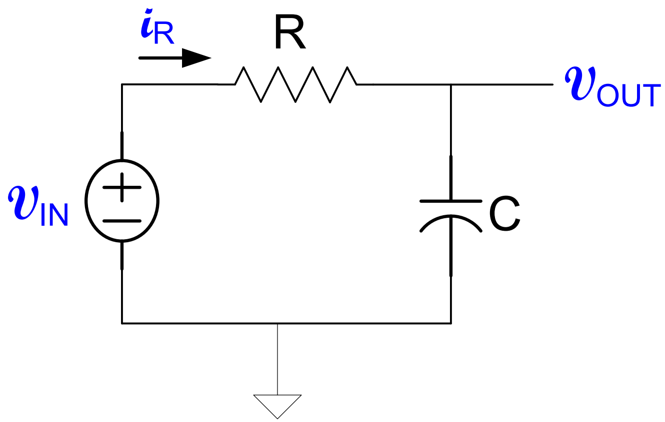
D(s) is characteristic of the dead network and is independent of where the excitation is applied or where the response is measured

D(s) is the same for ALL transfer functions of a given "dead network"

(if written in integer monic or unity constant form)

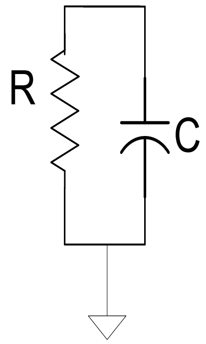
# Dead Networks

Example:



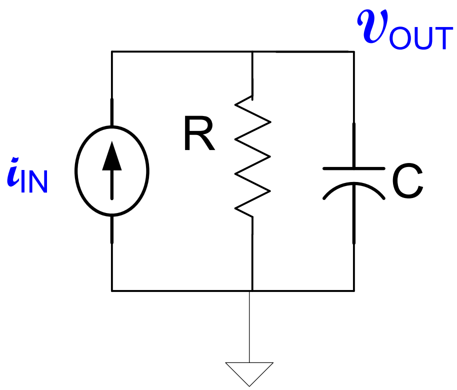

$$T(s) = \frac{1}{1+RCs}$$

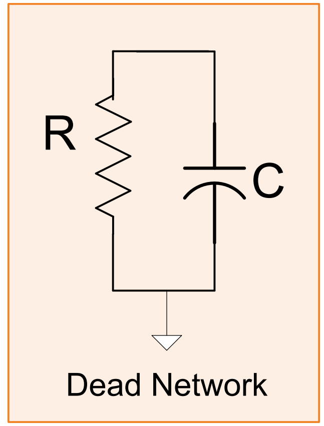
$$D(s) = 1+RCs$$




Dead Network

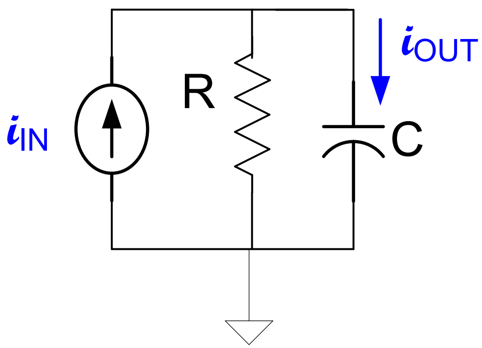
# Dead Networks

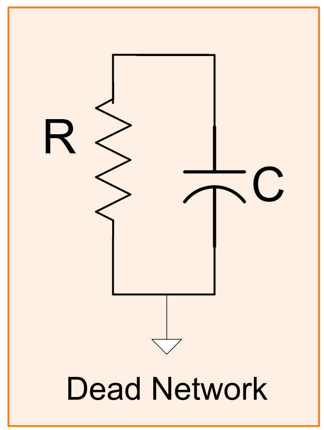



$$\frac{v_{OUT}}{i_{IN}} = T(s) = \frac{R}{1+RCs}$$

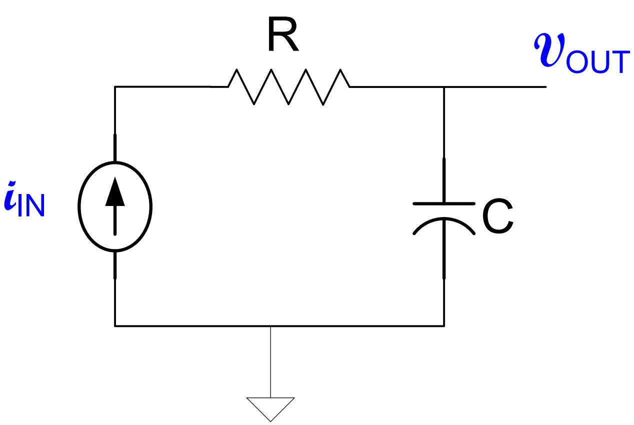
$$D(s) = 1+RCs$$

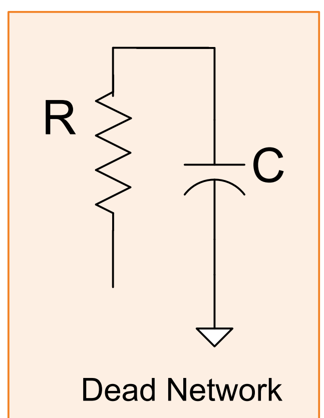



$$\frac{i_{OUT}}{i_{IN}} = T(s) = \frac{RCs}{1+RCs}$$

$$D(s) = 1+RCs$$



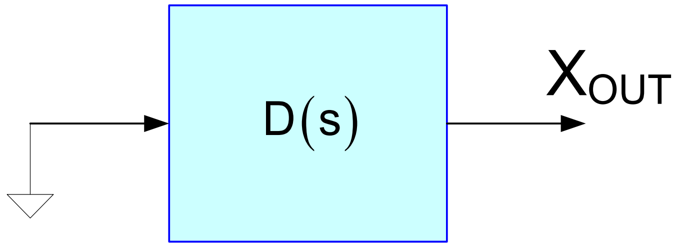




$$\frac{v_{OUT}}{i_{IN}} = T(s) = \frac{1}{Cs}$$

$$D(s) = Cs$$

Note: This has a different dead network!

D(s) is the same for ALL transfer functions of a given “dead network”



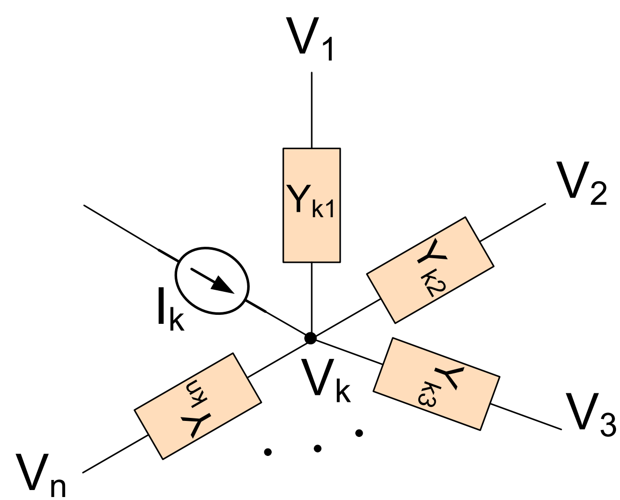

This is an important observation. Why is it true?

Plausibility argument:

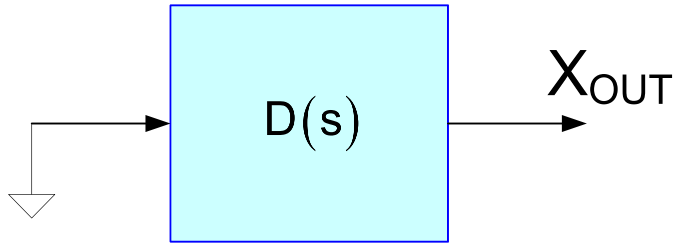
Consider a network with only admittance elements and independent current sources

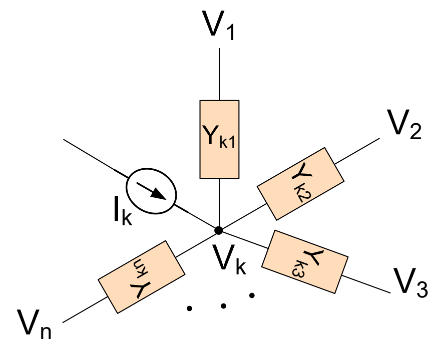
At node k, can write the equation

$$\sum_{\substack{i=1 \\ i \neq k}}^{n} Y_{ki}\left(V_k - V_i\right) = I_k$$



D(s) is the same for ALL transfer functions of a given “dead network”




Plausibility argument:

Doing this at each node results in the set of equations

$$\begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2n} \\ . & & & \\ . & & & \\ Y_{n1} & Y_{n2} & \cdots & Y_{nn} \end{bmatrix} \bullet \begin{bmatrix} V_1 \\ V_2 \\ . \\ . \\ . \\ V_n \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ . \\ . \\ . \\ I_n \end{bmatrix}$$

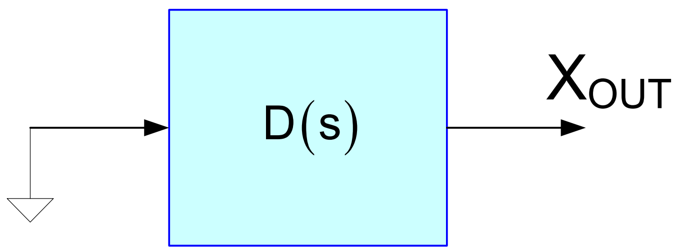
In matrix form

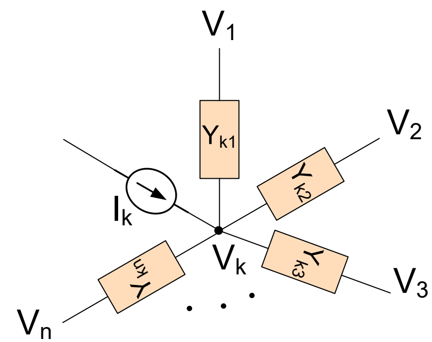
$$\mathbf{Y} \bullet \mathbf{V} = \mathbf{I}$$

The nodal voltages are given by

$$\mathbf{V} = \mathbf{Y}^{-1} \bullet \mathbf{I}$$

D(s) is the same for ALL transfer functions of a given "dead network"




Plausibility argument:

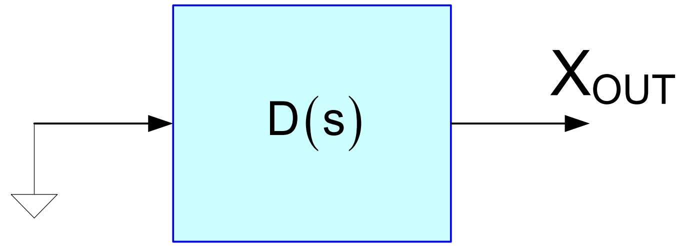
$$\mathbf{V} = \mathbf{Y}^{-1} \bullet \mathbf{I}$$

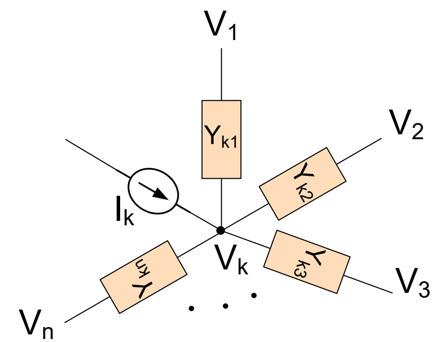
The nodal voltage $V_k$ in this solution is given by the ratio of two determinates where the one in the numerator is obtained by replacing the kth column with the excitation vector and the one in the denominator is the determinate of the indefinite admittance matrix **Y**

$$V_k = \frac{\begin{vmatrix} Y_{11} & Y_{12} & ..I_1. & Y_{1n} \\ Y_{21} & Y_{22} & ..I_2. & Y_{2n} \\ . & & & \\ . & & & \\ Y_{n1} & Y_{n2} & ..I_n. & Y_{nn} \end{vmatrix}}{\begin{vmatrix} Y_{11} & Y_{12} & ..... & Y_{1n} \\ Y_{21} & Y_{22} & ..... & Y_{2n} \\ . & & & \\ . & & & \\ Y_{n1} & Y_{n2} & ..... & Y_{nn} \end{vmatrix}}$$

Note the denominator is the same for all nodal voltages and is independent of the excitations – that is, it is dependent only upon the dead network

D(s) is the same for ALL transfer functions of a given “dead network”




Plausibility argument:

Note the denominator is the same for all nodal voltages and is independent of the excitations – that is, it is dependent only upon the dead network

Thus all branch voltages and all branch currents have the same denominator and this (after multiplying through by the correct power of s to make $V_k$ a rational fraction) is the characteristic polynomial D(s)

This concept can be extended to include independent voltage sources as well as dependent sources

$$V_k = \frac{\begin{vmatrix} Y_{11} & Y_{12} & ..I_1. & Y_{1n} \\ Y_{21} & Y_{22} & ..I_2. & Y_{2n} \\ . & & & \\ . & & & \\ Y_{n1} & Y_{n2} & ..I_n. & Y_{nn} \end{vmatrix}}{\begin{vmatrix} Y_{11} & Y_{12} & ..... & Y_{1n} \\ Y_{21} & Y_{22} & ..... & Y_{2n} \\ . & & & \\ . & & & \\ Y_{n1} & Y_{n2} & ..... & Y_{nn} \end{vmatrix}}$$

# Filter Concepts and Terminology

- 2-nd order polynomial characterization
- Biquadratic Factorization
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- → Root Characterization
- Scaling, normalization, and transformation

From previous lecture

# 2-nd order polynomial characterization

$\{a,b\}$ $\{\omega_o, Q\}$ $\{\zeta, \omega_o\}$ $\{p_1, p_2\}$

$\{\alpha, \beta\}$ $\{r, \theta\}$

Alternate equivalent parameter sets

Widely used interchangeably

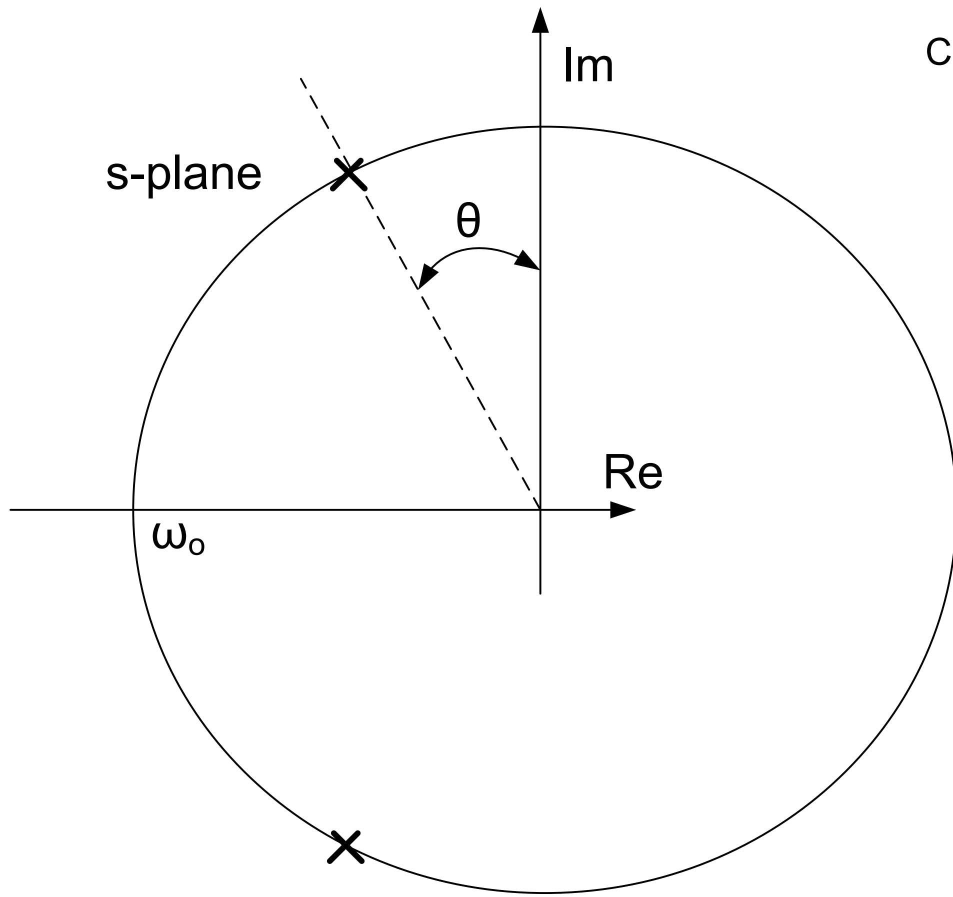
Easy mapping from one to another

Defined irrespective of whether polynomial appears in numerator or denominator of transfer function

If order is greater than 2, often multiple root pairing options so these parameter sets will not be unique for a given polynomial or transfer function

If cc roots exist, these will almost always be paired together (unique)

# Root characterization in s-plane
## (for complex-conjugate roots)




Characterization Parameters

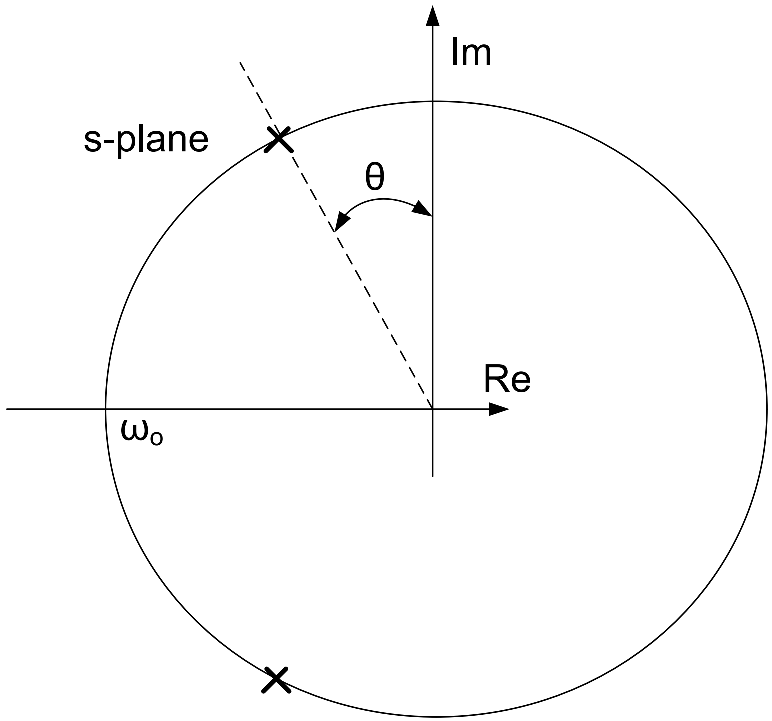
$\{\omega_0, Q\}$

$$s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2$$

1-1 relationship between angle θ and Q of root

For low Q,   θ is large
For high Q,   θ is small

# Root characterization in s-plane
## (for complex-conjugate roots)




$$s^2 + s\left(\frac{\omega_0}{Q}\right) + \omega_0^2$$

for θ=45°, Q=1/√2

for θ=90°, Q=1/2

roots located at $$s = -\frac{\omega_0}{2Q} \pm \frac{1}{2}\sqrt{\left(\frac{\omega_0}{Q}\right)^2 - 4\omega_0^2} = \omega_0\left(-\frac{1}{2Q} \pm \frac{1}{2}\sqrt{\left(\frac{1}{Q}\right)^2 - 4}\right)$$

for Q>0.5 the roots have an imaginary component

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{4Q^2 - 1}}\right)$$

# Filter Concepts and Terminology

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# Scaling, Normalization and Transformations

- Frequency scaling
- Frequency Normalization
- Impedance scaling
- Transformations
  - LP to BP
  - LP to HP
  - LP to BR

# Scaling, Normalization and Transformations

Frequency normalization: $$s_n = \frac{s}{\omega_0}$$

Frequency scaling: $$s = \omega_0 s_n$$

Purpose:

$\omega_0$ independent approximations

$\omega_0$ independent synthesis

Simplifies analytical expressions for T(s)

Simplifies component values in synthesis

Use single table of normalized filter circuits for given normalized approximating function

Note: The normalization subscript “n” is often dropped

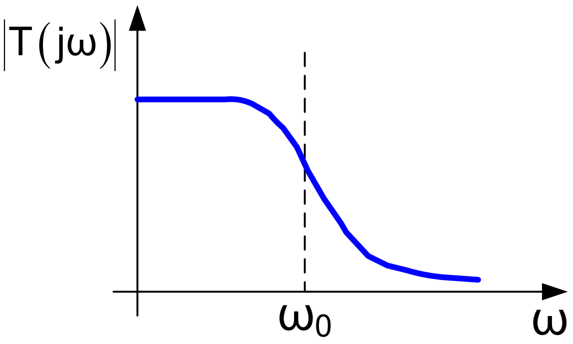
# Frequency normalization/scaling example

$$T(s) = \frac{6000}{s + 6000}$$

Define $\omega_0$=6000

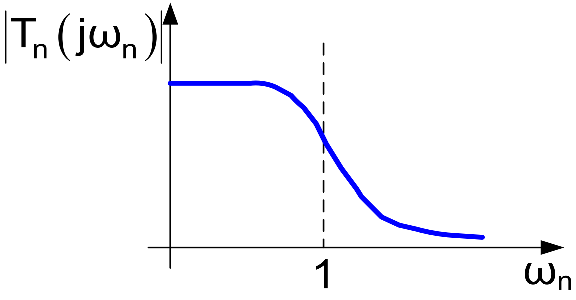
$$s_n = \frac{s}{\omega_0}$$

$$T(s) = \frac{\omega_0}{s + \omega_0}$$



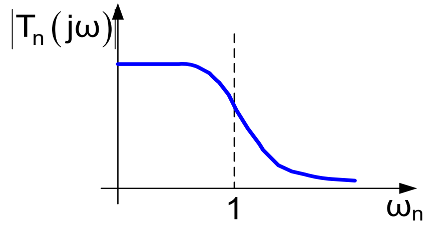

Normalized transfer function:

$$T_n(s_n) = \frac{1}{s_n + 1}$$

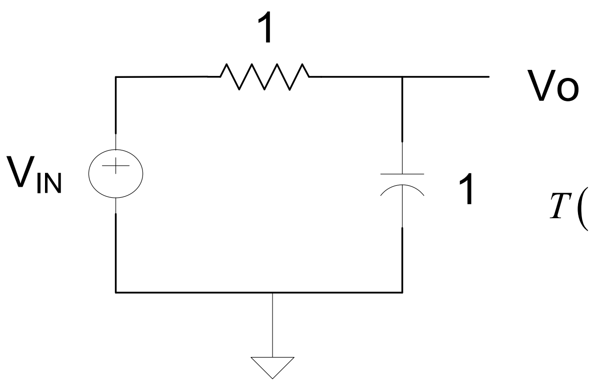


# Frequency normalization/scaling example

$$T_n(s_n) = \frac{1}{s_n + 1}$$



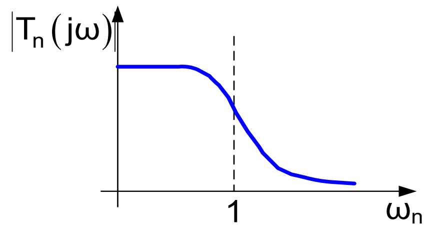

Synthesis of normalized function




$$T(s) = \frac{1}{s+1}$$

# Frequency normalization/scaling example

$$T_n(s_n) = \frac{1}{s_n + 1}$$



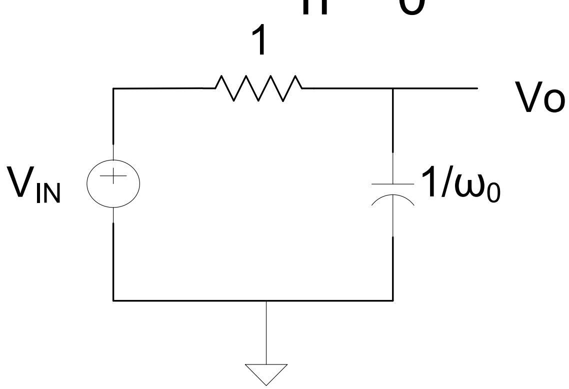

Frequency scaling transfer function by $\omega_0$

$$s = \omega_0 s_n$$

$$T(s) = \frac{1}{\left(\frac{s}{\omega_0}\right) + 1} \rightarrow T(s) = \frac{\omega_0}{s + \omega_0}$$

Frequency scaling circuit by $\omega_0$ (actually magnitude of $\omega_0$) (scale all energy storage elements in circuit)

$$C = C_n/\omega_0$$



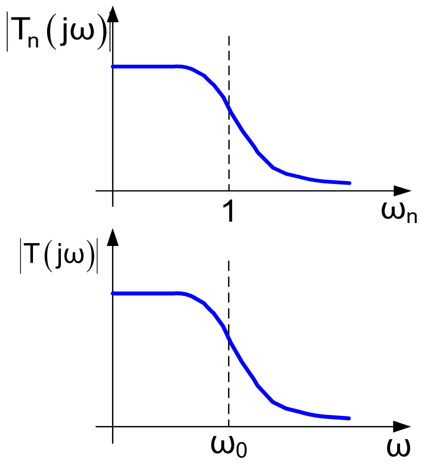

$$\rightarrow T(s) = \frac{\omega_0}{s + \omega_0}$$

Frequency scaled transfer function is that of the frequency scaled circuit !

# Frequency normalization/scaling example

$$T_n(s_n) = \frac{1}{s_n + 1}$$

$$T(s) = \frac{\omega_0}{s + \omega_0}$$




Frequency scaling / normalization does not change the shape of the transfer function, it only scales the frequency axis linearly

The frequency scaled circuit can be obtained from the normalized circuit simply by scaling the frequency dependent impedances (up or down) by the scaling factor

This makes the use of filter design tables for the design of lowpass filters practical whereby the circuits in the table all have a normalized band edge of 1 rad/sec.

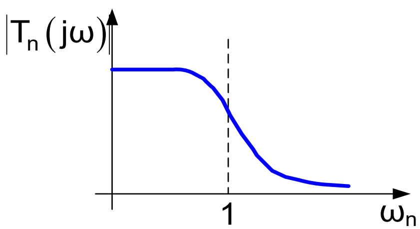
Anatol I. Zverev

# *Handbook of FILTER SYNTHESIS*

# Frequency normalization/scaling

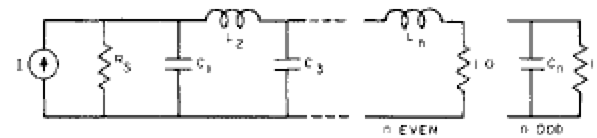
Example: Table for passive LC ladder Butterworth filter with 3dB band edge of 1 rad/sec and resistive source/load terminations

$$T_n(s_n) = \frac{1}{s_n + 1}$$




LOW PASS ELEMENT VALUES

BUTTERWORTH RESPONSE

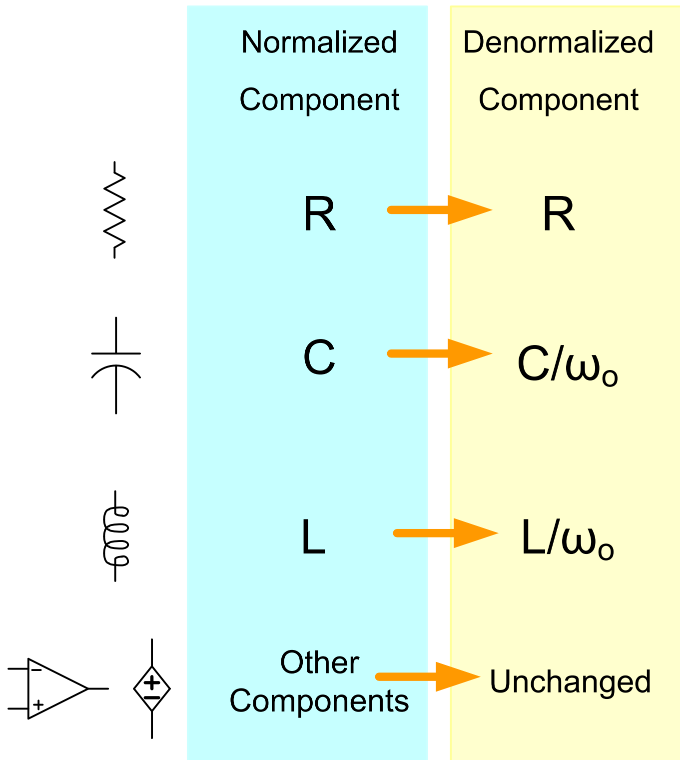



| n | $R_S$ | $C_1$ | $L_2$ | $C_3$ | $L_4$ |
|---|---|---|---|---|---|
| 2 | 1.0000 | 1.4142 | 1.4142 | | |
| | 1.1111 | 1.0353 | 1.9352 | | |
| | 1.2500 | 0.8485 | 2.1213 | | |
| | 1.4286 | 0.6971 | 2.4387 | | |
| | 1.6667 | 0.5657 | 2.8284 | | |
| | 2.0000 | 0.4483 | 3.3461 | | |
| | 2.5000 | 0.3419 | 4.0951 | | |
| | 3.3333 | 0.2447 | 5.3126 | | |
| | 5.0000 | 0.1557 | 7.7067 | | |
| | 10.0000 | 0.0743 | 14.8138 | | |
| | INF. | 1.4142 | 0.7071 | | |
| 3 | 1.0000 | 1.0000 | 2.0000 | 1.0000 | |
| | 0.9000 | 0.8082 | 1.6332 | 1.5994 | |
| | 0.8000 | 0.8442 | 1.3840 | 1.9259 | |
| | 0.7000 | 0.9152 | 1.1652 | 2.2774 | |
| | 0.6000 | 1.0225 | 0.9650 | 2.7024 | |
| | 0.5000 | 1.1811 | 0.7789 | 3.2612 | |
| | 0.4000 | 1.4254 | 0.6042 | 4.0642 | |
| | 0.3000 | 1.8380 | 0.4396 | 5.3634 | |
| | 0.2000 | 2.6687 | 0.2842 | 7.9102 | |
| | 0.1000 | 5.1672 | 0.1377 | 15.4554 | |
| | INF. | 1.5000 | 1.3333 | 0.5000 | |
| 4 | 1.0000 | 0.7654 | 1.8478 | 1.8478 | 0.7654 |
| | 1.1111 | 0.4657 | 1.5924 | 1.7439 | 1.4690 |
| | 1.2500 | 0.3983 | 1.6946 | 1.5110 | 1.8109 |
| | 1.4286 | 0.3251 | 1.8618 | 1.2913 | 2.1752 |
| | 1.6667 | 0.2693 | 2.1029 | 1.0824 | 2.6131 |
| | 2.0000 | 0.2175 | 2.4524 | 0.8826 | 3.1868 |
| | 2.5000 | 0.1692 | 2.9858 | 0.6911 | 4.0094 |
| | 3.3333 | 0.1237 | 3.8826 | 0.5072 | 5.1381 |
| | 5.0000 | 0.0804 | 5.6835 | 0.3307 | 7.9397 |
| | 10.0000 | 0.0392 | 11.0942 | 0.1616 | 15.6421 |
| | INF. | 1.5307 | 1.5772 | 1.0824 | 0.3827 |
| n | $1/R_S$ | $L_1$ | $C_2$ | $L_3$ | $C_4$ |

# Frequency normalization/scaling

The frequency scaled circuit can be obtained from the normalized circuit simply by scaling the frequency dependent impedances (up or down) by the scaling factor

Component denormalization by factor of $\omega_0$




**Component values of energy storage elements are scaled down by a factor of $\omega_0$**

# Desgin Strategy

Theorem: A circuit with transfer function T(s) can be obtained from a circuit with normalized transfer function $T_n(s_n)$ by denormalizing all frequency dependent components.

$C \rightarrow C/\omega_o$

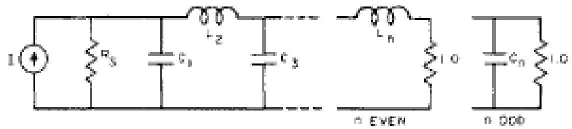
$L \rightarrow L/\omega_o$

Example: Design a V-V passive 3rd-order Lowpass Butterworth filter with a 3-db band-edge of 1K rad/sec and equal source and load terminations.

(from the BW approximation which will be discussed later:) $T(s)=\frac{1}{s^3+2s^2+2s+1}$

LOW PASS ELEMENT VALUES

BUTTERWORTH RESPONSE

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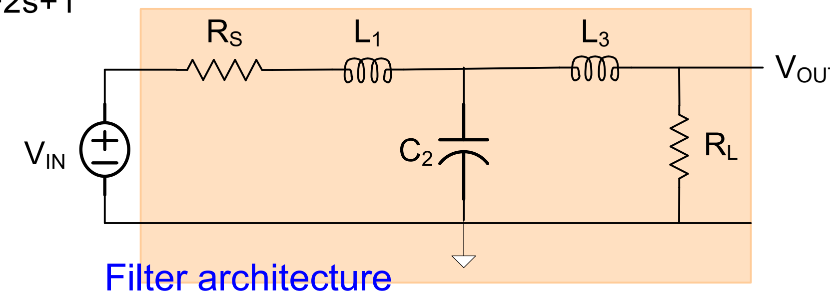
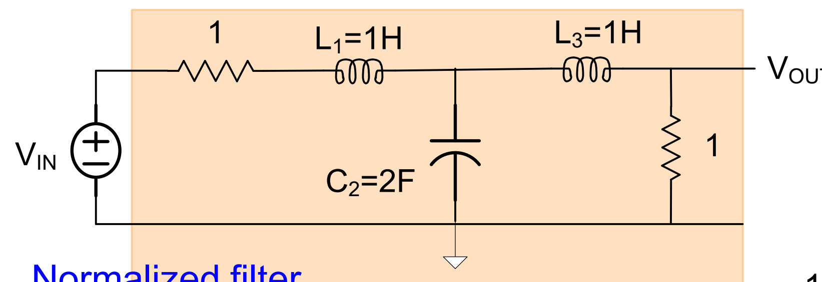
| | 1.6667 | 0.5657 | 2.8284 | | |
| | 2.0000 | 0.4483 | 3.3461 | | |
| | 2.5000 | 0.3419 | 4.0951 | | |
| | 3.3333 | 0.2447 | 5.3126 | | |
| | 5.0000 | 0.1557 | 7.7067 | | |
| | 10.0000 | 0.0743 | 14.8138 | | |
| | INF. | 1.4142 | 0.7071 | | |
| 3 | 1.0000 | 1.0000 | 2.0000 | 1.0000 | |
| | [illegible] | [illegible] | [illegible] | [illegible] | |
| | 0.8000 | 0.8442 | 1.3840 | 1.9259 | |
| | 0.7000 | 0.9152 | 1.1652 | 2.2774 | |
| | 0.6000 | 1.0225 | 0.9650 | 2.7024 | |
| | 0.5000 | 1.1811 | 0.7789 | 3.2612 | |
| | 0.4000 | 1.4254 | 0.6042 | 4.0642 | |
| | 0.3000 | 1.8380 | 0.4396 | 5.3634 | |
| | 0.2000 | 2.6687 | 0.2842 | 7.9102 | |
| | 0.1000 | 5.1672 | 0.1377 | 15.4554 | |
| | INF. | 1.5000 | 1.3333 | 0.5000 | |
| 4 | 1.0000 | 0.7654 | 1.8478 | 1.8478 | 0.7654 |
| | 1.1111 | 0.4657 | 1.5924 | 1.7439 | 1.4690 |
| | 1.2500 | 0.3982 | 1.6946 | 1.5110 | 1.8109 |
| | 1.4286 | 0.3251 | 1.8618 | 1.2913 | 2.1752 |
| | 1.6667 | 0.2693 | 2.1029 | 1.0824 | 2.6131 |
| | 2.0000 | 0.2175 | 2.4524 | 0.8826 | 3.1868 |
| | 2.5000 | 0.1692 | 2.9858 | 0.6911 | 4.0094 |
| | 3.3333 | 0.1237 | 3.8826 | 0.5072 | 5.3381 |
| | 5.0000 | 0.0804 | 5.6835 | 0.3307 | 7.9397 |
| | 10.0000 | 0.0392 | 11.0942 | 0.1616 | 15.6421 |
| | INF. | 1.5307 | 1.5772 | 1.0824 | 0.3827 |
| n | $1/R_S$ | $L_1$ | $C_2$ | $L_3$ | $C_4$ |


Filter architecture

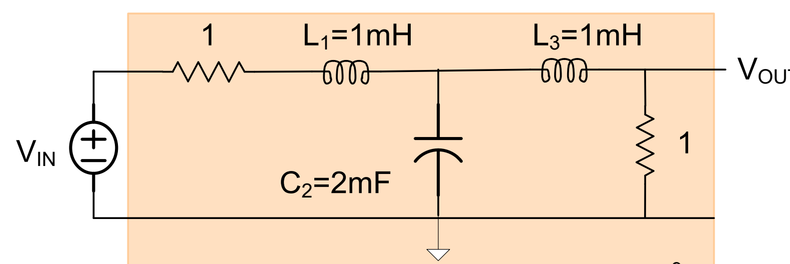



Normalized filter

$$T(s)=K\frac{1}{s^3+2s^2+2s+1}$$

C → C/θ

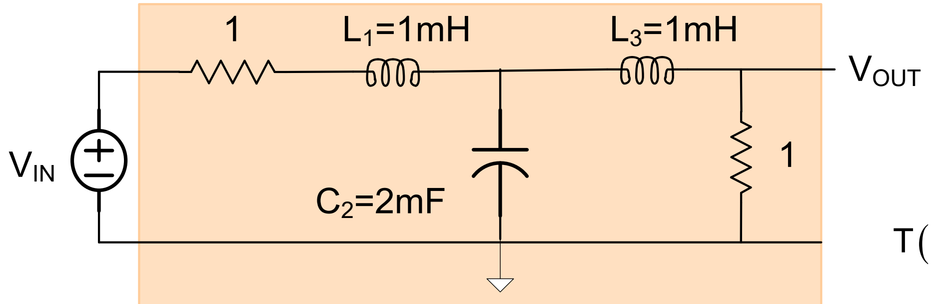
L → L/θ




Denormalized filter

$$T(s)=K\frac{10^9}{s^3+2\bullet 10^3 s^2+2\bullet 10^6 s+10^9}$$

Example: Design a V-V passive 3rd-order Lowpass Butterworth filter with a band-edge of 1K Rad/Sec and equal source and load terminations.




$$T(s)=K\frac{10^9}{s^3+2\bullet 10^3 s^2+2\bullet 10^6 s+10^9}$$

Is this solution practical?

**Some component values are too big and some are too small !**

# Filter Concepts and Terminology

- Frequency scaling
- Frequency Normalization
- Impedance scaling
- Transformations
  - LP to BP
  - LP to HP
  - LP to BR

# Impedance Scaling

Impedance scaling of a circuit is achieved by multiplying ALL impedances in the circuit by a constant

$R \rightarrow \theta R$

$C \rightarrow C/\theta$

$L \rightarrow L\theta$

$A \rightarrow$

- $\theta A$ for transresistance gain
- $A$ for dimensionless gain
- $A/\theta$ for transconductance gain

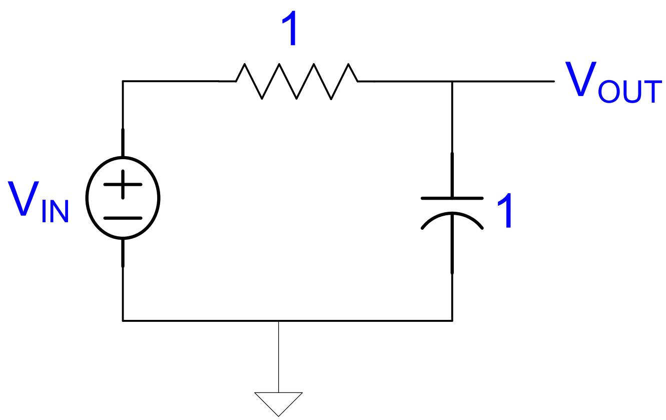
# Impedance Scaling

Theorem: If all impedances in a circuit are scaled by a constant $\theta$, then

a) All dimensionless transfer functions are unchanged
b) All transresistance transfer functions are scaled by $\theta$
c) All transconductance transfer functions are scaled by $\theta^{-1}$

# Impedance Scaling

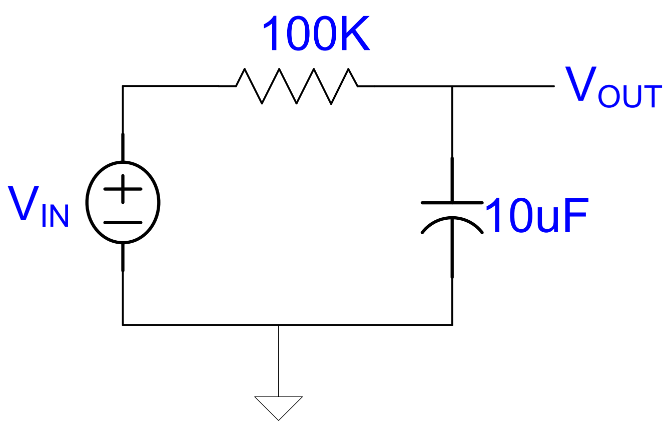
Example:




$$T(s) = \frac{1}{s+1}$$

T(s) is dimensionless

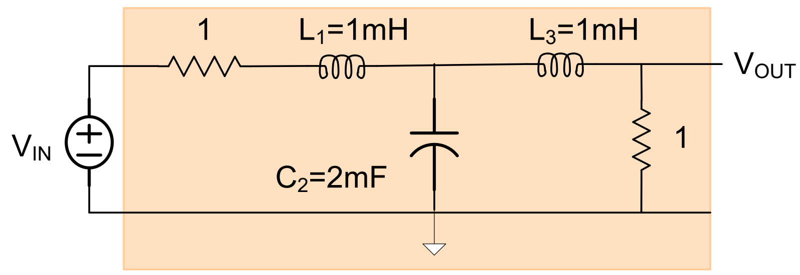
Impedances scaled by $\theta=10^5$




$$T(s) = \frac{1}{s+1}$$

Note second circuit much more practical than the first

Example: Design a V-V passive 3rd-order Lowpass Butterworth filter with a band-edge of 1K Rad/Sec and equal source and load terminations.




$$T(s)=K\frac{10^9}{s^3+2\bullet10^3s^2+2\bullet10^6s+10^9}$$

Is this solution practical?

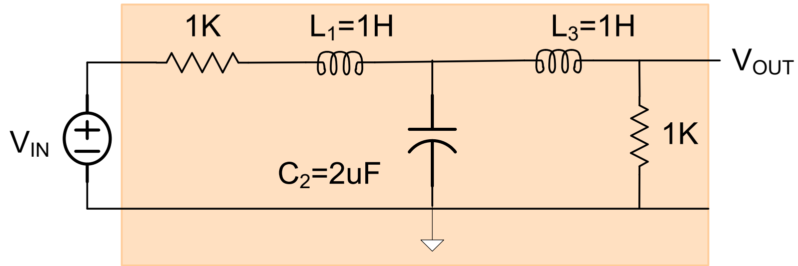
**Some component values are too big and some are too small !**

Impedance scale by θ=1000

$R \rightarrow \theta R$

$C \rightarrow C/\theta$

$L \rightarrow \theta L$




$$T(s)=K\frac{10^9}{s^3+2\bullet10^3s^2+2\bullet10^6s+10^9}$$

Component values more practical

# Transformations

–LP to BP

–LP to HP

–LP to BR

It can be  shown the standard HP, BP, and BR approximations can be obtained by a frequency transformation of a standard LP approximating function

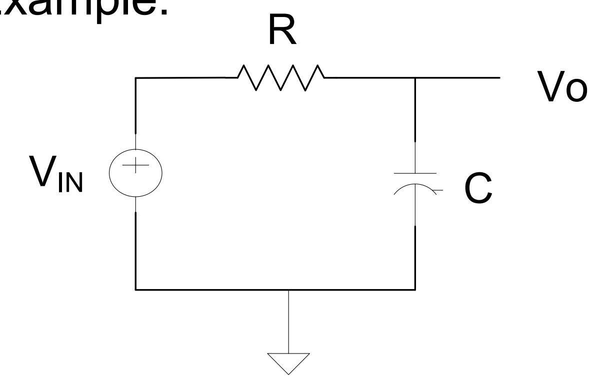
Will address the LP approximation first, and then provide details about the frequency transformations

# Typical approach to lowpass filter design

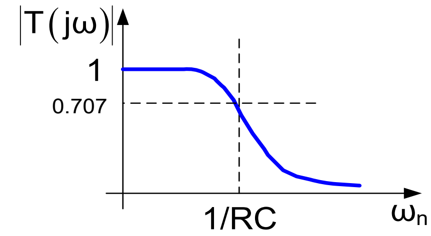
1. Obtain normalized approximating function
2. Synthesize circuit to realize normalized approximating function
3. Denormalize circuit obtained in step 2
4. Impedance scale to obtain acceptable component values

# Degrees of Freedom

Example:




$$T(s) = \frac{V_O}{V_{IN}} = \frac{1}{RCs+1}$$




Circuit has two design variables: {R,C}

One key controllable performance characteristic of this circuit: $\omega_0 = \frac{1}{RC}$

(there could be others such as total area, magnitude of impedance,…)

If $\omega_0$ is specified for a design, circuit has
(and nothing else is specified)

2 design variables
1 constraint

1 Degree of Freedom

Performance/Cost strongly affected by how degrees of freedom in a design are used !

# Degrees of Freedom

The number of degrees of freedom in the design of a system is the difference between the total number of design variables and the number of constraints for the design.

Important to recognize the number of degrees of freedom available in a design and the number of constraints.

- If the number of design variables is less than the number of constraints in a specific system, the system is over-constrained

- Even if the number of degrees of freedom is greater than or equal to 1, a solution may not exist

Stay Safe and Stay Healthy !

End of Lecture 5